Estimation of viscous and fatigue dissipation of bituminous concrete in repeated loading tests

Rajneesh Gupta
Indian Institute of Technology Madras, Chennai, India

Atul Narayan S. P.
Indian Institute of Technology Madras, Chennai, India

ABSTRACT: Dissipation is a powerful tool for characterizing fatigue behaviour and has been employed for various materials. But the use of dissipation in characterizing fatigue of bituminous concrete is complicated by the presence of other modes of dissipation than fatigue, particularly dissipation due to viscous friction. The challenge is therefore to separate fatigue dissipation due to repeated loading from viscous dissipation. Some methods have already been suggested the same including the use of the dissipated pseudo-strain energy (DPSE) and the ratio of dissipated energy change (RDEC).

In this study, the two primary methods available for separating fatigue dissipation, DPSE and RDEC, were analysed using principles of thermodynamics and experimental observations in the literature. It was found that while they are useful in certain specific cases, these methods cannot be universally used in all laboratory and field tests.

Accurate estimation of fatigue and viscous dissipation requires accurate characterization of the viscoelastic behaviour of damaged bituminous concrete. However, fatigue of bituminous concrete occurs at higher stress levels. It is difficult to conduct experiments to elicit the pure viscoelastic behaviour at such stress levels as experiments at high stress levels tend to damage the specimen. It is therefore necessary to make some constitutive assumptions about the viscoelastic behaviour of damaged bituminous concrete. One such constitutive assumption was used in this study to develop an alternative method to estimate fatigue dissipation. Fatigue dissipation was estimated using this method for bituminous concrete prepared with one unmodified binders and two binders modified by a plastomer and an elastomer.

1 INTRODUCTION

For the past two decades, the main focus of research with respect to fatigue of bituminous concrete has been to develop a model describing the fatigue behaviour of bituminous mixtures that is consistent with the experimental results, particularly a model that can describe the fatigue behaviour in both load-controlled and displacement-controlled fatigue tests. While many a framework has been proposed to realise these objectives, the approach used by many, possibly because of its simplicity, is the energy dissipation approach. Fatigue damage occurs mainly through formation of new surfaces by cracking, which is in turn caused by loss of cohesion within the bituminous mastic, and the loss of adhesion between aggregates and the mastic. These mechanisms of fatigue-related changes to the internal structure of bituminous mixtures involve dissipation of energy. The dissipation through these mechanisms of damage is often referred to as the “fatigue dissipation”. In repeated loading tests in uniaxial condition, the total dissipation over a cycle when the strain in reasonably small is given by

$$W = \oint \sigma(t)\dot{\epsilon}(t)dt \tag{1}$$

where σ and ϵ are the axial stress and strain, respectively. By measuring the accumulation of dissipation in repeated loading tests, the extent of damage in the material can be estimated. This forms the basis of the dissipation approach to characterise fatigue and has been used to successfully characterize the fatigue behaviour of various materials (Inglis 1927, Coffin Jr 1957, Feltner & Morrow 1961). Since mechanical dissipation can be easily measured in repeated loading tests, this approach promises a simple and easy method to characterise fatigue behaviour of bituminous mixtures in such tests.

The use of dissipation framework in characterizing fatigue behaviour of bituminous mixtures, however, is complicated to an extent by the presence of other modes of dissipation such as that due to viscous friction, in addition to that due to fatigue damage. Fatigue dissipation needs to be separated from other modes of dissipation before it can be used for characterizing fatigue of bituminous mixtures. The methods proposed in the literature for this purpose can be broadly classified into two groups: use of a dissipated pseudo-strain energy (DPSE) for characterization of fatigue and the use of a parameter known as the ratio of dissipated energy change (RDEC) to characterize fatigue. These two approaches are analyzed in this study the details of which are presented in the next few sections. A new approach for estimating fatigue dissipation is then proposed as an alternative. This is followed by an evaluation of the proposed approach by applying it to certain experimental results.

2 ANALYSIS OF EXISTING DISSIPATION BASED APPROACHES

2.1 *Pseudo-strain energy dissipation approach*

Kim & Little (1989) was one of the first to propose the use of pseudo-strain energy dissipation to characterizing fatigue of bituminous concrete. Kim & Little (1989) proceeded by defining a new strain quantity called the pseudo-strain, ϵ_R in the following manner. For a particular strain history, the corresponding stress history when there is no fatigue damage and the response is purely viscoelastic can be determined from the constitutive equations for the viscoelastic material. The stress history corresponding to the purely viscoelastic response divided by a suitably chosen reference modulus parameter, G_R, was defined as the pseudo-strain, ϵ_R. For instance, when a sinusoidal strain of the form

$$\epsilon(t) = \epsilon_0 \sin \omega t, \tag{2}$$

is applied on a linear viscoelastic material, the corresponding sinusoidal stress response when there is no damage is given by

$$\sigma_{ve}(t) = |G^*_{ve}|\epsilon_0 \sin(\omega t + \delta_{ve}), \tag{3}$$

where G^*_{ve} and δ_{ve} are the dynamic modulus and the phase angle of the viscoelastic material and the subscript ve is used to denote that the stress response is purely viscoelastic devoid of any damage. The pseudo strain in this case is defined in terms of σ_{ve} as

$$\epsilon_R(t) = \frac{\sigma_{ve}(t)}{G_R} = \epsilon_0 \frac{|G^*_{ve}|}{G_R} \sin(\omega t + \delta_{ve}). \tag{4}$$

Now, the pseudo-strain energy dissipation, W_{ps}, over a cycle is defined in terms of pseudo-strain as follows:

$$W_{ps} = \oint \sigma(t)\dot{\epsilon_R}(t)dt \tag{5}$$

where $\sigma(t)$ is the actual stress in the material. It can be seen that the stress σ experienced by the material is expected to be equal to σ_{ve} only when there is no damage to the material, and is expected to be different otherwise. Consequently, the pseudo-strain energy dissipation in a cycle is zero when there is no damage and is positive otherwise. In view of this, Kim & Little (1989) regarded the pseudo-strain energy dissipation as equal to the fatigue dissipation of the material and used it to characterise fatigue. Following their work, Lee et al. (2000), Kim et al. (2003), Lee et al. (2003), Masad et al. (2008) and Castelo Branco et al. (2008) further developed the approach and used pseudo-strain energy dissipation in crack-growth models, continuum-damage models, etc., to characterise fatigue.
The pseudo-strain dissipation energy in a displacement-controlled fatigue test is typically calculated in the following manner, as illustrated by Bhasin et al. (2009). In a displacement controlled fatigue test involving sinusoidal loading as shown in (2), the stress response will not be sinusoidal but would possess some other waveform. However, conventionally in the literature, it is approximated as a sinusoidal waveform, in which case the stress response would be approximately of the form

$$\sigma(t) = G_n \epsilon_0 \sin(\omega t + \delta_n), \tag{6}$$

where G_n and δ_n are parameters similar to the modulus and phase angle, respectively, that depend on the number of cycles of loading that has been applied on the material. For the DPSE approach, the viscoelastic part of the stress response is taken to be of the form

$$\sigma_{ve_n}(t) = G_n \epsilon_0 \sin(\omega t + \delta_{ve}), \tag{7}$$

where δ_{ve} is the phase angle measured for undamaged bituminous concrete. Accordingly, the pseudo-strain in the nth cycle is

$$\epsilon_{R_n}(t) = \epsilon_0 \frac{G_n}{G_R} \sin(\omega t + \delta_{ve}), \tag{8}$$

Now, the total dissipation W_{total_n}, the viscoelastic dissipation W_{ve_n} and the pseudo-strain energy dissipation W_{ps_n} in the nth cycle are given by

$$W_{total_n} = \oint \sigma_n(t)\dot{\epsilon}(t)dt = \pi G_n \epsilon_0^2 \sin\delta_n \tag{9}$$

$$W_{ve_n} = \oint \sigma_{ve}(t)\dot{\epsilon}(t)dt = \pi G_n \epsilon_0^2 \sin\delta_{ve} \tag{10}$$

$$W_{ps_n} = \oint \sigma_n(t)\dot{\epsilon}_{R_n}(t)dt = \pi \frac{G_n^2}{G_R} \epsilon_0^2 \sin(\delta_n - \delta_{ve}) \tag{11}$$

Bhasin et al. (2009) and others consider the pseudo-strain energy dissipation W_{ps_n} to be equivalent to the dissipation due to fatigue damage in the cycle. If viscous dissipation and fatigue dissipation are the only modes of dissipation of bituminous concrete, this assumption implies that the sum of viscous dissipation and pseudo strain energy dissipation should be equal to the total dissipation in the cycle. However,

$$W_{ps_n} + W_{ve_n} = \pi G_n \epsilon_0^2 \sin\delta_{ve} + \pi \frac{G_n^2}{G_R} \epsilon_0^2 \sin(\delta_n - \delta_{ve}) \tag{12}$$

$$= \pi G_n \epsilon_0^2 \left(\sin\delta_{ve} + \pi \frac{G_n}{G_R} \sin(\delta_n - \delta_{ve}) \right) \tag{13}$$

$$= \pi G_n \epsilon_0^2 \left(\left[\frac{G_n}{G_R} \cos\delta_{ve} \right] \sin\delta_n + \left[1 - \frac{G_n}{G_R} \cos\delta_n \right] \sin\delta_{ve} \right) \tag{14}$$

which can never be equal to W_{total_n} for all possible values of δ_n for any choice of G_R, unless δ_{ve} is zero or δ_n is equal to δ_{ve}. Moreover, when G_R is assumed to be equal to G_n and δ_n is taken to be always greater than or equal to δ_{ve}, as it is usually done (Bhasin et al. 2009),

$$W_{total_n} = \pi G_n \epsilon_0^2 \sin\delta_n \tag{15}$$

$$= \pi G_n \epsilon_0^2 [\sin\delta_{ve} \cos(\delta_n - \delta_{ve}) + \sin(\delta_n - \delta_{ve}) \cos\delta_{ve}]. \tag{16}$$

Since, both δ_{ve} and $\delta_n - \delta_{ve}$ are both expected to be such that

$$0 \le \delta_{ve}, \delta_n - \delta_{ve} \le \frac{\pi}{2}, \tag{17}$$

an inequality can be obtained for W_{total_n}:

$$W_{total_n} \le \pi G_n \epsilon_0^2 [\sin\delta_{ve} + \sin(\delta_n - \delta_{ve})], \tag{18}$$

$$W_{total_n} \le W_{ve_n} + W_{ps_n}, \tag{19}$$

with equality occurring only when $\delta_{ve} = 0$ or $\delta_n = \delta_{ve}$. Thus, the standard assumptions for reference modulus and the phase angle parameter results in a condition where the sum of viscous dissipation and pseudo strain energy dissipation is greater than the total dissipation. For any other assumptions for the reference modulus, total dissipation being equal to the sum of viscous dissipation and pseudo-strain energy dissipation is not guaranteed for all δ_n. Since the fatigue dissipation W_{fd_n} would be such that

$$W_{total_n} = W_{ve_n} + W_{fd_n}, \tag{20}$$

in the absence of other modes of dissipation, it can be inferred that the pseudo-strain energy dissipation is not the same as fatigue dissipation.

2.2 *Rate of dissipation approach*

The second approach, initially suggested by Ghuzlan & Carpenter (2000) and later developed by Carpenter et al. (2003), Shen & Carpenter (2005) and Carpenter & Shen (2006), involves the use of the rate of dissipation for characterisation of fatigue. Ghuzlan & Carpenter (2000) argued that in repeated loading tests on bituminous concrete, the viscous dissipation remains constant in all the cycles and that the change in dissipation with each cycle is only due to fatigue damage. Consequently, the rate of change of dissipation with cycles of load repetition would correspond to the rate of damage in the material. Thus, by measuring the rate of dissipation with cycles, the viscous dissipation can be eliminated from total dissipation. Ghuzlan & Carpenter (2000) used a parameter called Ratio of Dissipation Energy Change (RDEC) to quantify the extent of damage in bituminous concrete. This parameter is defined as follows:

$$RDEC_n = \frac{DE_{n+1} - DE_n}{DE_n}, \tag{21}$$

where DE_n is the total dissipation in the n^{th} cycle. In a typical sinusoidal fatigue test, Ghuzlan & Carpenter (2000) found this RDEC parameter to initially start with a relatively high value and decrease with number of repetitions at a steep rate until it reaches a minimum. This is followed by a period in which RDEC remains constant with number of repetitions which Ghuzlan & Carpenter (2000) identified as the plateau period. Failure in both load-controlled and displacement-controlled tests is marked by a rapid increase in RDEC with the number of repetitions. For most bituminous concrete, the plateau period is the predominant part of the RDEC vs. number of repetitions curve. Ghuzlan & Carpenter (2000) used the RDEC in the plateau called the plateau value (PV) for characterisation of fatigue of bituminous concrete. Typically, the value of RDEC in the plateau period is not a constant but varies over a certain range and the value of RDEC at the point where the stiffness of bituminous concrete decreases to 50% of the initial stiffness is taken as the plateau value (PV).

The key assumption made in the rate of dissipation approach is that any change in total dissipation between successive cycles is only due to fatigue damage. However, in many displacement-controlled fatigue tests, it has been observed that the total dissipation in a cycle may decrease with successive repetitions (see Figure 1) and yet the material may suffer fatigue damage as evidenced by the decrease in the dynamic modulus with the progress of the fatigue test (Kim et. al. 2003, Yoo & Al-Qadi, 2010). RDEC measured in such tests would be negative over a large portion of the test. The change in dissipation between cycles will be due to both change in the viscous dissipation characteristics and fatigue damage. Thus, the assumption that damage occurs only when the dissipation increases with load repetitions may not be valid for all fatigue tests.

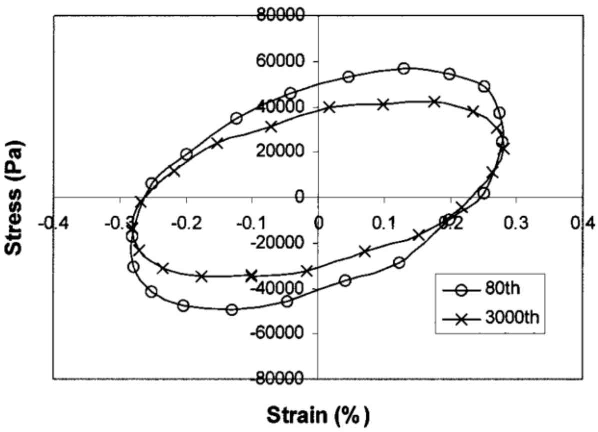

Figure 1. A sample displacement-controlled fatigue test result with total dissipation in a cycle decreasing with each cycle (Source: Kim et. al. (2003))

3 AN ALTERNATIVE APPROACH TO ESTIMATE FATIGUE DISSIPATION

In this study, a new approach for estimating fatigue dissipation for bituminous concrete is proposed. The approach involves making a suitable constitutive assumption on the viscoelastic behaviour of damaged bituminous concrete, essentially an assumption relating the viscoelastic behaviour on the extent of damage. In any fatigue test, the constitutive assumptions can be used to determine the viscous dissipation in each cycle as the test progresses. Since the total dissipation can be calculated from the stress and strain measurements, the fatigue dissipation can also be determined.

The constitutive assumption made in this study is that the total strain in bituminous concrete is the sum of a viscoelastic strain and another strain part, and the viscoelastic strain response of damaged bituminous concrete is the same as the purely viscoelastic strain response of undamaged bituminous concrete when subjected to the same stress loading. This assumption implies that in a load-controlled fatigue test with the stress of the form

$$\sigma(t) = \sigma_0 \sin \omega t, \tag{22}$$

the total strain response can be represented by

$$\epsilon(t) = \frac{\sigma_0}{G_n} \sin(\omega t - \delta_n). \tag{23}$$

and the viscoelastic strain component is as per the assumption

$$\epsilon_{ve_n}(t) = \frac{\sigma_0}{|G_{ve}^*|}\sin(\omega t - \delta_{ve}). \tag{24}$$

The corresponding viscous dissipation is

$$W_{ve_n} = \oint \sigma(t)\dot{\epsilon}_{ve_n}(t)dt = \pi\frac{\sigma_0^2}{|G_{ve}^*|}\sin\delta_{ve}. \tag{25}$$

In a displacement controlled fatigue test with the strain of the form shown in (2), the stress response is as shown in (6).The viscoelastic strain in this case is given by

$$\epsilon_{ve}(t) = \frac{\sigma_n}{|G_{ve}^*|}\sin(\omega t - \delta_{ve}), \tag{26}$$

where σ_n is the amplitude of the stress in the nth cycle. Using the expression for $\sigma(t)$ from (6), the above equation can be rewritten as

$$\epsilon_{ve_n}(t) = \frac{G_n}{|G_{ve}^*|}\epsilon_0\sin(\omega t + \delta_n - \delta_{ve}), \tag{27}$$

The corresponding viscous dissipation is given by

$$W_{ve_n} = \oint \sigma_n(t)\dot{\epsilon}_{ve_n}(t)dt = \pi\frac{G_n^2}{|G_{ve}^*|}\epsilon_0^2\sin\delta_{ve}. \tag{28}$$

In both cases, the fatigue dissipation can be determined by subtracting viscous dissipation from the total dissipation:

$$W_{fd_n} = W_{total_n} - W_{ve_n}. \tag{29}$$

The constitutive assumption is essentially equivalent to assuming that fatigue damage does not change the viscous dissipation characteristics of bituminous concrete.

4 EXPERIMENTAL PROCEDURE

Three bitumen were used in the study - one unmodified bitumen conforming to VG 30 grade as per the Indian standard IS 73, a bitumen modified by a plastomer and another modified by elastomer, both conforming to PMB 40 grade as per IS 15462. Locally available granite aggregates with a gradation adhering to BC-Grade 2 gradation specified by MORT&H 2013 specifications (Ministry Of Road Transport and Highways 2013) was used for the mix. 5% binder content was used for all the bitumen. Mixing and conditioning were conducted as per AASHTO R30 and compaction was carried out using a PReSBOX shear compactor manufactured by IPC, Australia. Beams were cast with a target air voids of $4 \pm 0.5\%$. From compacted beams of dimensions $450 \times 150 \times 145 - 185$mm, three cylindrical specimen of diameter 93 ± 0.5 mm and height 150 ± 0.5 mm were cored out. Cylindrical specimens not meeting the air void requirement of $4 \pm 0.5\%$ were discarded.

Fatigue tests were conducted in uniaxial tension using an Asphalt Mixture Performance Tester (AMPT) manufactured by IPC, Australia. The specimen was glued to two steel plates with a suitable epoxy adhesive to apply tension. Three linear variable differential transducers (LVDT's) of gauge length 70 mm and a range of ± 0.5 mm were fixed onto the specimen, all oriented along the axial direction and separated at $120°$ angle from each other. The deformation measured by the LVDT's were averaged and divided by the gauge length to obtain the axial strain of the specimen. All the fatigue tests were conducted in load-controlled mode with a haversine loading of 10 Hz frequency. All the tests were conducted at a temperature of 20 °C.

5 RESULTS AND DISCUSSION

Tests were conducted in the following manner. Initially small amplitude haversine stress loading was applied on the specimen to measure the dynamic modulus $|G_{ve}^*|$ and the phase angle δ_{ve} of the bituminous concrete at 10 Hz. Followed by this, a larger amplitude haversine loading at 10 Hz frequency was applied in addition to a seating load of 0.1 kN until complete failure of the sample. For the mixtures with the two modified bitumen a stress amplitude of 550 kPa was applied, and for the mixture with unmodified bitumen a stress amplitude of 515 kPa was applied during the fatigue test. The dynamic modulus, phase angle of the bituminous concrete specimens at 10 Hz frequency, the amplitude of the applied stress for the fatigue tests, the corresponding viscous dissipation in a cycle and the number of repetitions until complete failure are presented in Table 1.

Table 1. Fatigue testing results

Binder used in mixture	**Unmodified**	**Plastomer modified**	**Elastomer modified**
Loading frequency (Hz)	10	10	10
Stress amplitude (kPa)	515	550	550
Dynamic Modulus (MPa)	17089	15907	10981
Phase angle (Degrees)	15.93	13.03	22.14
Viscous dissipation per cycle (J/m^3)	3.34	3.37	8.15
Number of cycles to failure	105400	173000	152500

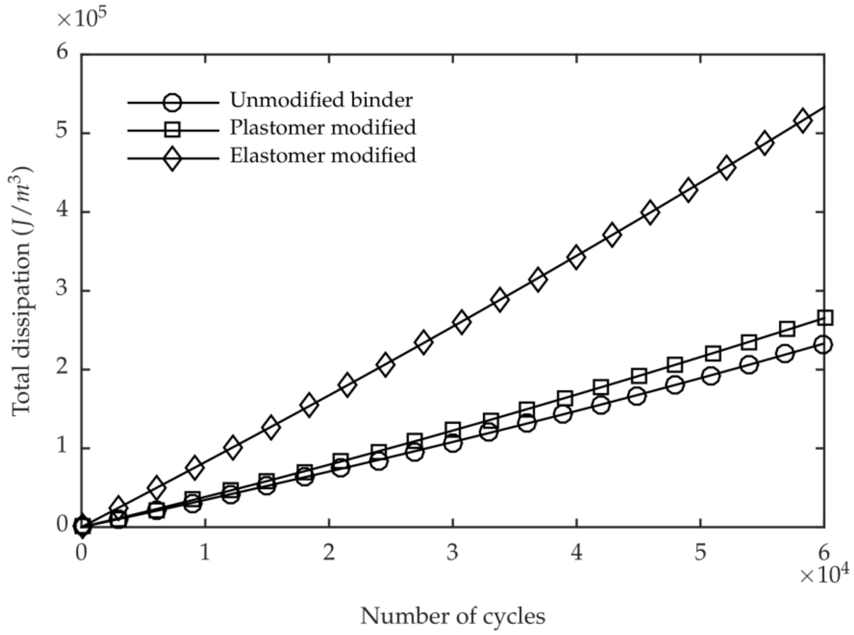

Figure 2. Variation of cumulative total dissipation with the number of cycles of repetition for the bituminous mixtures

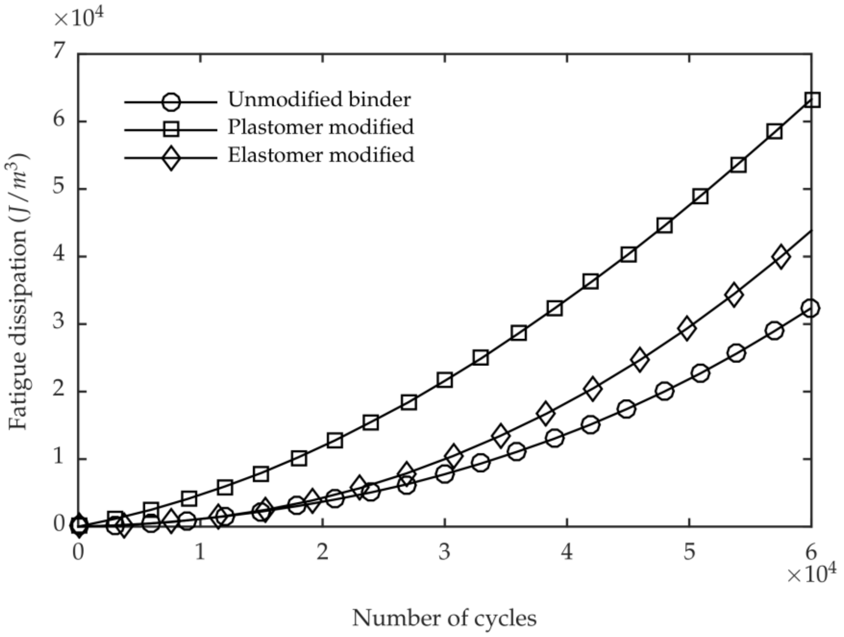

Figure 3. Variation of cumulative fatigue dissipation with the number of cycles of repetition for the bituminous mixtures

Using the measured strain along with the applied stress, the total dissipation in each cycle was calculated by numerically integrating equation (25). The LVDT's recorded the axial deformation only till the deformation exceeded their range. The total dissipation therefore could be calculated only until that point. The cumulative total dissipation thus estimated for each bituminous concrete is plotted against the number of load repetitions in Figure 2. The total dissipation vs. number of repetitions curve appear to be nearly a linear function for all the bitminous concrete. The total dissipation was highest for the bituminous concrete with the elastomer modified binder and least for the bituminous concrete with the unmodified binder.

According to the constitutive assumptions made in this study, the viscous dissipation does not change with the cycles of load repetition in a stress-controlled test. The viscous dissipation in a cycle was calculated using equation (25). The fatigue dissipation in each cycle was then determined by subtracting the viscous dissipation from the total dissipation. The cumulative fatigue dissipation was determined by summing over the fatigue dissipation estimated for each cycle. The cumulative fatigue dissipation vs. the number of cycles of repetition for all the bituminous concrete are presented in Figure 3. At about 50,000 cycles, the total cumulative dissipation for the bituminous concrete with elastomer modified binder is 4.35×10^5 J/m^3, while the cumulative fatigue dissipation is 2.9×10^4 J/m^3, only 6.67% of the total dissipation. The value for bituminous mixtures with plastomer modified binder and unmodified binder are 22% and 11%, respectively. Thus, a significant part of total dissipation is due to viscous friction rather than fatigue dissipation.

The cumulative fatigue dissipation versus number of load repetitions was fitted by a second order polynomial of the form $aN + bN^2$, where N is the number of cycles of load repetition. This quadratic form was found to completely fit the cumulative fatigue dissipation curve, with the R^2 value of the fits being higher than 0.99 for all the tests. Such a quadratic form of the variation of cumulative fatigue dissipation with the number of cycles of load repetition implies that the fatigue dissipation in a cycle increases linearly with the number of cycles of load repetition. Fatigue dissipation in load-controlled tests can therefore be characterized by two constants.

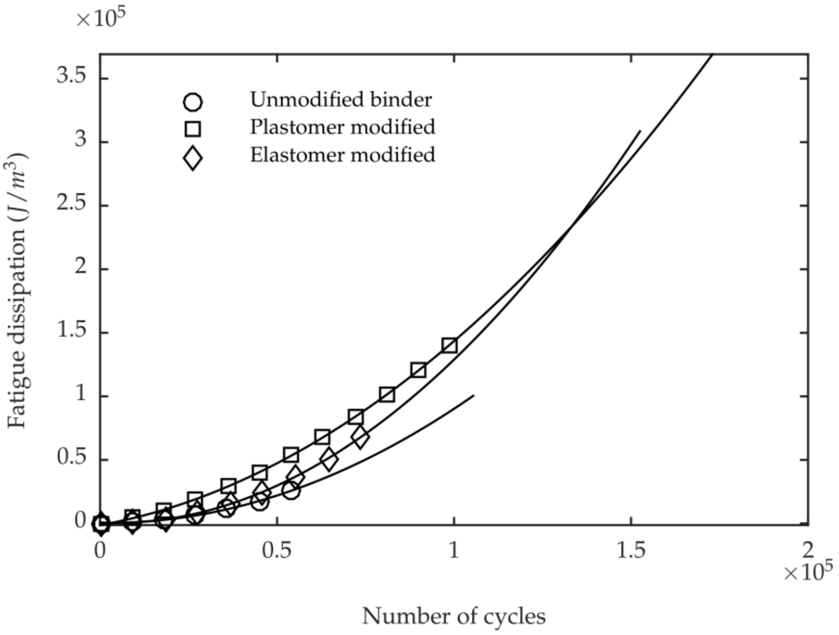

Figure 4. Quadratic polynomial t to the cumulative fatigue dissipation curves and extended up to the point of failure

Assuming the quadratic variation would continue until failure of the specimen, the cumulative fatigue dissipation was predicted until the end of the test for the three bituminous mixtures and are presented in Figure 4. The results show that the bituminous concrete with the plastomer modified bitumen dissipated the maximum energy through fatigue damage until complete failure. This was followed by the bituminous concrete with the elastomer modified bitumen. The bituminous concrete with the unmodified bitumen dissipated the least energy through fatigue damage before failure. This order could be indicative of the resistance to fatigue failure of the bitumen tested in this study.

6 CONCLUSION

Dissipation in repeated loading tests of bituminous concrete can be due to both viscous friction and fatigue damage. Approaches existing in the literature to separate fatigue dissipation from viscous dissipation, particularly the pseudo-strain energy dissipation approach and the rate of dissipation approach, were analyzed in the light of experimental observations in fatigue tests. Some issues and shortcomings were found in both the approaches. While they could be useful in certain circumstances, both methods cannot be universally applied for all fatigue tests.

A new approach to estimate fatigue dissipation in fatigue tests was proposed. The base of this approach is a constitutive assumption on the viscoelastic behaviour of damaged bituminous concrete. Once such a constitutive assumption is in place, the viscous dissipation in a cycle can be determined and can be used in the estimation of fatigue dissipation. In this study, it was assumed that the viscoelastic response of damaged bituminous concrete would be such that the viscous dissipation characteristics do not change with the extent of damage. Fatigue dissipation was estimated in this manner for three different bituminous concrete from load-controlled fatigue tests. It was found that a considerable portion of the total dissipation in fatigue tests is due to viscous friction rather than fatigue damage.

The next step in the development of the new approach would be to validate the constitutive assumption. Validation would require the measurement of viscoelastic characteristics of damaged bituminous concrete. Measurements should be taken at different levels of damage to obtain the evolution of viscoelastic behaviour with fatigue damage.

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